Linear stability analysis of evolving thin shell wormholes

Institute of Theoretical Physics
Beijing University of Technology (BJUT), China
Email: lac@emails.bjut.edu.cn

李爱晨 (graduate)

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brief introduction to thin-shell wormholes

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Ai-Chen Li, Wu-Long Xu and Ding-Fang Zeng[1]

Theoretical Physics Division, College of Applied Sciences, Beijing University of Technology, 100 Ping Le Yuan, ChaoYang District, Beijing, P.R. China

E-mail: lac@emails.bjut.edu.cn, wlxu@emails.bjut.edu.cn, dfzeng@bjut.edu.cn

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Abstract. Using ideas of cosmological perturbation theories, we make general linear stability analysis of dynamic thin-shell wormholes constructed by cutting-and-pasting two building-block spacetimes at arbitrary joining shell radius. We observed that in appropriate parameter choices, dynamical thin shell wormholes following from such a cut-and-paste procedure can be kept stable during the whole evolution process towards the final extremal point on which the joining shell radius reaches on static values. Our work forms a valuable complemen-

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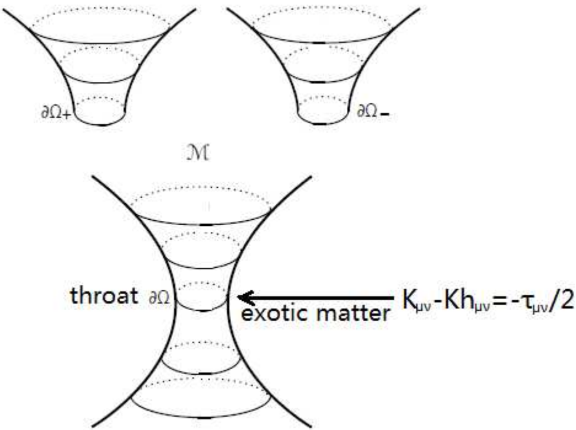
dynamic thin-shell wormholes constructed by cutting-and-pasting two building-spacetime



brief introduction to thin-shell wormholes

The motion of thin-shell in the static bulk:

$$ds^2_{bulk} = g_{MN}dx^M dx^N = -A(r)dT^2 + B(r)dr^2 + R^2(r)d\Omega^2_{n-2,k}$$

Bulk	T	r	$\vec{X}_{n-2}$
thin shell	t	$\times$	$\vec{x}_{n-2}$
embedding	$T(t)$	$r(t)$	$\vec{x}_2 = \vec{X}_{n-2}$

Table: Coordinates parameterizing the whole bulk spacetime and the thin shell in it.The induced metric for moving thin shell is obtained according to $h_{\mu\nu} = e^A_\mu e^B_\mu g_{AB}$, where $e^M_\mu = \frac{\partial X^M}{\partial x^\mu}$

$$ds^2_{shell} = h_{\mu\nu}dx^\mu dx^\nu = -dt^2 + R^2[r(t)]d\Omega^2_{n-2}$$

Exact FLRW-metric!

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For simplification, n=4, k=1

$$ds^2_{bulk} = -f(r)dT^2 + f(r)^{-1}dr^2 + r^2 d\Omega_2^2$$

$$f(r) = \begin{cases} 1 - \frac{2m}{r} \pm \frac{r^2}{\ell^2}(AdS/dS) \\ 1 - \frac{2mr^2}{r^3+2ml^2}(Hayward) \end{cases}$$

Setting the energy-momentum tensor in the $\partial\Sigma$ as simple ideal fluid types,

$$\tau^{\mu}_{\nu} = Diag\{-\rho, P, P\}$$

The evolution of thin-shell wormholes could be described by following equations:

$$Darmois - Israel \ : \begin{cases} dr/r = -d\rho/2(\rho + \phi(\rho)) \\ \dot{r}^2 = H(r)\ ,\ H(r) = \rho^2(r)r^2/16 - f(r) \end{cases}$$

$$Equation\ of\ state \ : \ P = \phi(\rho)$$

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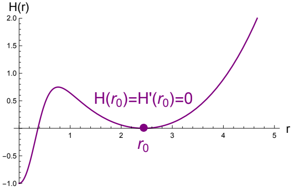
Previous perturbation method:
E. Poisson and M. Visser, *Phys. Rev. D* **52 (1995) 7318**

$$[\delta\dot{r}(t)]^2 = H[r_0 + \delta r(t)] - H[r_0]$$
$$= \frac{1}{2}H''[r_0][\delta r(t)]^2 + O[\delta r(t)^3]$$



Stability $\Longrightarrow H''[r_0] < 0$

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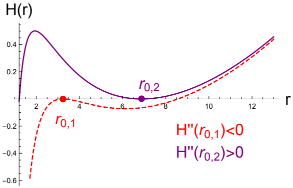
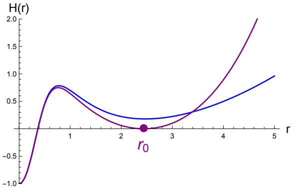
Shortage of previous perturbation method:

(i)Only born-static wormholes are stable. Natural?



(ii)How to judge the stability of ever-evolving wormholes?



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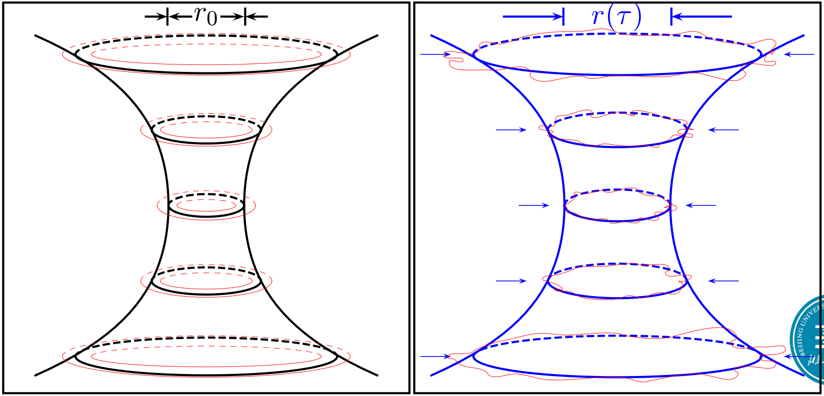
Shortage of previous perturbation method:

(iii)Quantum mechanically, the inhomogeneous and anisotropy fluctuations are unavoidable.

How to describe these fluctuations around the joining shells?



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Fluctuations around the joining shell with unperturbed bulk:
P. Brax, D. Langlois and M. Rodriguez-Martinez, *Phys. Rev. D* **67 (2003) 104022**
(i)perturbations of geometrical quantities

$$X^A(x^\mu) = \bar{X}^A(x^\mu) + \zeta(x)\bar{n}^A$$

$$\begin{cases} h_{\mu\nu} = \bar{h}_{\mu\nu} + 2\zeta\bar{\mathcal{K}}_{\mu\nu} \\ n^A = \bar{n}^A + \alpha\bar{n}^A + \beta^\mu \frac{\partial \bar{X}^A}{\partial x^\mu} \end{cases} \begin{cases} \alpha = -\frac{1}{2}\zeta\bar{n}^M\bar{n}^N\bar{n}^C\partial_C\bar{g}_{MN} \\ \beta^\mu = -\bar{h}^{\mu\nu}(\partial_\nu\zeta + \zeta\bar{e}^B_\nu\bar{n}^C\bar{n}^A\partial_C\bar{g}_{AB}) \end{cases}$$

$$\Longrightarrow$$

$$\begin{aligned} \delta\mathcal{K}_{\tau\tau} = & -\ddot{\zeta} + \dot{\zeta}\bar{u}^B[\bar{n}^A\nabla_{\bar{A}}\bar{n}_B - \bar{n}^C\bar{n}^N\partial_C\bar{g}_{BN}] + \zeta\bar{g}^{AL}\bar{u}^N\bar{u}^B\nabla_{\bar{N}}\bar{n}_L(\nabla_{\bar{A}}\bar{n}_B + \nabla_{\bar{B}}\bar{n}_A) \\ & + \zeta\bar{u}^A\bar{u}^B[\bar{\Gamma}^M_{AB}\bar{n}_M\bar{n}^Q\bar{n}^C\partial_C\bar{n}_Q - \bar{n}^C\bar{n}^M\partial_A\bar{n}_B\partial_C\bar{n}_M - \frac{1}{2}\bar{n}^N\bar{n}^C\partial_C\bar{\Gamma}_{NAB} \\ & - \frac{1}{2}\bar{n}_Q\bar{\Gamma}_{NAB}\bar{n}^C\partial_C\bar{g}^{NQ} - \bar{n}^N\partial_C\bar{g}_{BN}\partial_A\bar{n}^C + \frac{1}{2}\bar{n}^N\partial_C\bar{g}_{AB}\partial_N\bar{n}^C] \\ \delta\mathcal{K}_{\tau i} = & -\partial_i\dot{\zeta} + \partial_i\zeta[\frac{1}{2}\bar{n}^A\bar{u}^B\nabla_{\bar{A}}\bar{n}_B - \frac{1}{2}\bar{n}^C\bar{u}^B\bar{n}^N\partial_C\bar{g}_{BN}] + \frac{1}{2}\bar{u}^B\bar{h}^{kj}\partial_B\bar{g}_{ki}\partial_j\zeta \\ \delta\mathcal{K}_{ij} = & -\partial_i\partial_j\zeta + \frac{1}{2}\dot{\zeta}\bar{u}^N\partial_N\bar{g}_{ij} + \frac{1}{2}\zeta[\bar{g}^{kl}\bar{n}^Q\bar{n}^N\partial_Q\bar{g}_{jl}\partial_N\bar{g}_{ki} - \bar{n}^N\bar{n}^C\bar{n}^Q\partial_N\bar{g}_{ij}\partial_C\bar{n}_Q \\ & + \bar{n}^N\partial_C\bar{g}_{ij}\partial_N\bar{n}^C + \bar{n}^C\bar{n}^N\partial_C\partial_N\bar{g}_{ij} + \bar{n}^C\bar{n}_Q\partial_N\bar{g}_{ij}\partial_C\bar{g}^{NQ}] \end{aligned}$$

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Fluctuations around the joining shell with unperturbed bulk:
(ii)perturbed energy momentum tensor

$$\delta\tau_\tau^\tau = -\delta\rho$$
$$\delta\tau_i^\tau = R(1+\omega)\rho\partial_i v$$
$$\delta\tau_j^i = \delta p\delta_j^i + \delta\pi_j^i \ , \ \delta\pi_j^i = \partial^i\partial_j\delta\pi - \frac{1}{2}\delta_j^i\partial^k\partial_k\delta\pi$$

(iii)consider the perturbed junction condition which relates matters and geometry

$$\delta\mathcal{K}_\nu^\mu = -\frac{1}{2}(\delta\tau_\nu^\mu - \frac{1}{2}\delta\tau\delta_\nu^\mu) \Longrightarrow \begin{cases} \delta\mathcal{K}_\tau^\tau = \frac{1}{2}(\frac{n-3}{n-2}\delta\rho + \delta P) \\ \delta\mathcal{K}_i^\tau = -\frac{1}{2}R(\bar{\rho}+\bar{P})\partial_i v \\ \delta\mathcal{K}_j^i = \begin{cases} -\frac{\kappa_5^2}{2}(\partial^i\partial_j\delta\pi) & (i \neq j) \\ -\frac{\kappa_5^2}{2}\delta\rho & (i = j \ with \ sum) \end{cases} \end{cases}$$

combine the expression δP , $\delta\rho$ with perturbed equations of state $\delta P = \phi'(\rho)\delta\rho$

$$\ddot{\eta}_{\ell m} + b[\phi'(\rho(r)) \ , \ f(r) \ , \ \dot{r} \ , \ r]\dot{\eta}_{\ell m} + c[\phi'(\rho(r)) \ , \ f(r) \ , \ \dot{r} \ , \ r \ , \ \ell]\eta_{\ell m} = 0$$
$$where \ \zeta(\tau,\theta,\varphi) = \sum_{\ell m}\eta_{\ell m}(\tau)Y_{\ell m}(\theta,\varphi)$$

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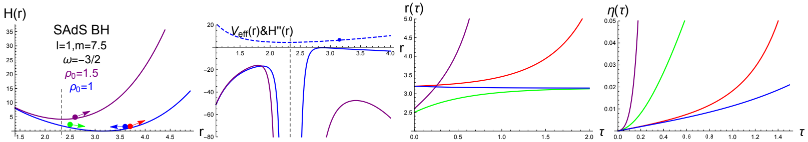
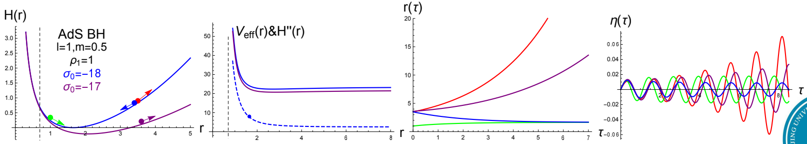
AdS spacetime & Phantom like state $P = \omega\rho$, $\omega < 0$:



AdS spacetime & Chaplygin gas state $P\rho = \sigma_0\rho$, $\sigma_0 < 0$:



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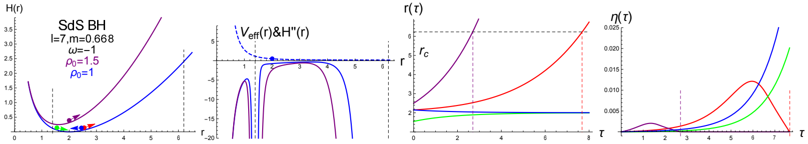
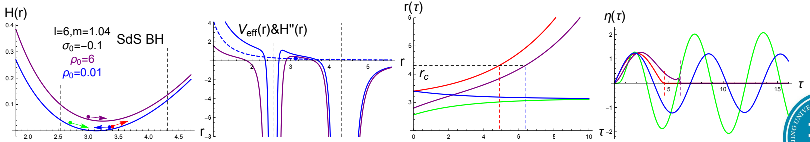
dS spacetime & Phantom like state $P = \omega\rho$, $\omega < 0$:



dS spacetime & Chaplygin gas state $P\rho = \sigma_0\rho$, $\sigma_0 < 0$:



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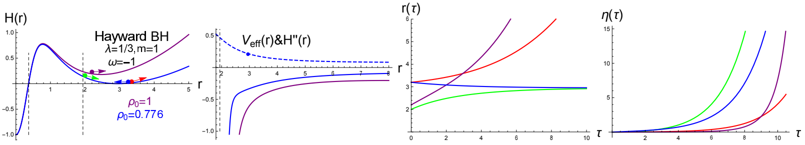
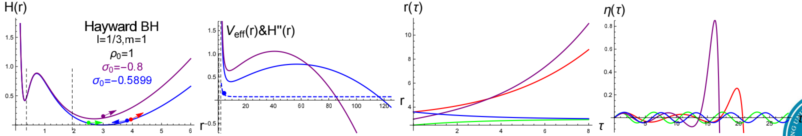
Hayward spacetime & Phantom like state $P = \omega\rho$, $\omega < 0$:



Hayward spacetime & Chaplygin gas state $P\rho = \sigma_0\rho$, $\sigma_0 < 0$:



Thank you!